00 P1 A6	**5A** $\bullet^1$ $g = 2k,\ f = -k,\ c = -k - 2$ (stated $\bullet^2$ $r^2 = 5k^2 + k + 2$ $\bullet^3$ $(real\ \ r \Rightarrow)\ 5k^2 + k + 2 > 0$ $\bullet^4$ $\Delta = 1 - 40 = -39$ $\bullet^5$ minimum t.p. so radius exists for all k.
00 P2 A2	**4C, 3C , 2C** $\bullet^1$ midpoint $= (-1, 5)$ $\bullet^2$ $m_{PQ} = \frac{9-1}{1-(-3)}$ $\bullet^3$ $m_{\perp} = -\frac{1}{2}$ $\bullet^4$ $y - 5 = -\frac{1}{2}(x - (-1))$ ans: $x + 2y = 9$ $\bullet^5$ $y_C = 4$ stated $\bullet^6$ $radius = 5\ \ or\ \ equiv.$ stated $\bullet^7$ $(x-1)^2 + (y-4)^2 = 25$ $\bullet^8$ $y = 9$ $\bullet^9$ $T = (-9, 9)$
01 P1 Q11	**4B, 3B, 3A** $\bullet^1$ $r_P = \sqrt{16 + 25 - 9} = \sqrt{32} = 4\sqrt{2}$ $\bullet^2$ $r_P + r_Q = 4\sqrt{2} + 2\sqrt{2} = 6\sqrt{2}$ $\bullet^3$ $C_P = (4, 5)$ $\bullet^4$ $C_PC_Q = \sqrt{6^2 + 6^2} = 6\sqrt{2}$ *and* "so touch" $\bullet^5$ $m_r = -1$ $\bullet^6$ $m_{tgt} = +1$ $\bullet^7$ $y - 1 = 1(x + 4)$ ans: $y = x + 5$ $\bullet^8$ $x^2 + (x+5)^2 - 8x - 10(x+5) + 9 = 0$ $\bullet^9$ $2x^2 - 8x - 16 = 0$ $\bullet^{10}$ $x = 2 \pm 2\sqrt{3}$
01 P2 Q7	**1C, 4C, 1C, 2B** $\bullet^1$ $x = 7$ $\bullet^2$ midpoint $= (5, 4)$ $\bullet^3$ $m_{AC} = \frac{2}{3}$ $\bullet^4$ $m_{\perp} = -\frac{3}{2}$ $\bullet^5$ $y - 4 = -\frac{3}{2}(x - 5)$ ans: $3x + 2y = 23$

	$\bullet^6$ $x=7,\quad y=1$ $\bullet^7$ $(x-7)^2+(y-1)^2$ $\bullet^8$ $(x-7)^2+(y-1)^2=26$
02 P1 Q1	**4C** $\bullet^1$ $C=(-1,1)$ $\bullet^2$ $m_{rad}=\frac{2}{3}$ $\bullet^3$ $m_{tgt}=-\frac{3}{2}$ $\bullet^4$ $y-3=-\frac{3}{2}(x-2)$ ans: $2y+3x=12$
7.(JAN) 02 P1	**4C, 2B** $\bullet^1$ centre $=(-2,5)$ $\bullet^2$ $m_{rad}=\frac{2}{-4}$ $\bullet^3$ $m_{tgt}=2$ $\bullet^4$ $y-3=2(x-2)$ ans: $y-2x=-1$ $\bullet^5$ $\frac{dy}{dx}=-2x+6$ $\bullet^6$ $m=2$ and complete **OR** $\bullet^5$ $-x^2+6x-5=2x-1$ $(x-2)^2=0$ $\bullet^6$ $\Rightarrow$ equal roots so tgt at $x=2$
5.(JAN) 02 P2	**7A, 2A** $\bullet^1$ $y=k-2x$ **stated** $\bullet^2$ $x^2+(k-2x)^2-2x-4=0$ $\bullet^3$ $5x^2-(2+4k)x+(k^2-4)=0$ $\bullet^4$ discriminant $=0$ **stated** $\bullet^5$ $(2+4k)^2-4\times5\times(k^2-4)=0$ $\bullet^6$ $-4k^2+16k+84=0$ $\bullet^7$ $k=7$ $\bullet^8$ $5x^2-30x+45=0$ $\bullet^9$ $(3,1)$
03 P1 Q11	**5C, 2A**

	•[1] $A=(12,-5)$ •[2] $OA=13$ •[3] $r_B=8$ •[4] $B=(24,0)$ •[5] $(x-24)^2+y^2=64$ •[6] $p=\frac{5}{144}$ •[7] $q=-24$
03 P2 Q4	**5C, 6B** •[1] $\frac{dy}{dx}=3x^{2\ldots}$ •[2] $\frac{dy}{dx}=3x^2+4x-3$ •[3] $m=\frac{dy}{dx}_{x=1}=4$ gradient stated •[4] $y_{x=1}=2$ •[5] $y-2=4(x-1)$ •[6] $y=4x-2$ •[7] $x^2+(4x-2)^2-12x-10(4x-2)+44=0$ •[8] $17x^2-68x+68=0$ •[9] $17(x-2)(x-2)=0$ •[10] equal roots $\Rightarrow$ tangent •[11] pt of contact $=(2,6)$
04 P2 Q8	**4C, 5C, 2C** •[1] $A(6,1)$ •[2] $m_{AP}=2$ *STATED* •[3] $m_{PQ}=-\frac{1}{2}$ •[4] $y+1=-\frac{1}{2}(x-5)$ *and complete* •[5] $x=3-2y$ •[6] $(3-2y)^2+y^2+10(3-2y)+2y+6=0$ •[7] $5y^2-30y+45=0$ •[8] *solve and get double root* $\Rightarrow$ *tangent* •[9] $5(y-3)^2=0$ •[10] $Q=(-3,3)$ •[11] $PQ=\sqrt{80}$ **OR** •[1] $(3-2y)^2+y^2-12(3-2y)-2y+32=0$ •[2] $5(y+1)^2=0$ •[3] *double root* $\Rightarrow$ *tangent* •[4] $x=3-2y=3-2\times(-1)=5$ **OR**

	•[8] $use\ discriminant, and\ get\ zero \Rightarrow tangent$ •[9] $b^2 - 4ac = (-30)^2 - 4.5.45 = 0$ **OR** •[8] $BP = 10\ units,\ BQ = radius = \sqrt{20}\ units$ •[9] $by\ Pythagoras\ PQ = \sqrt{80}$ **OR** •[5] $y = \frac{1}{2}(3 - x)$ •[6] $(x)^2 + \left(\frac{1}{2}(3-x)\right)^2 + 10(x) + 2\left(\frac{1}{2}(3-x)\right) + 6 = 0$ •[7] $5x^2 + 30x + 45 = 0$ •[8] $5(x+3)^2 = 0$ •[9] $double\ root \Rightarrow tangency$ $or\ \ b^2 - 4ac = 900 - 4.5.45 \Rightarrow tangency$ **OR** •[5] $centre\ B = (-5, -1)$ •[6] $diam: y + 1 = 2(x+5)$ •[7] $2x + 9 = \frac{3-x}{2}$ •[8] $Q = (-3, 3)$ •[9] $check: 9 + 9 - 30 + 6 + 6 = 0$
05 P1 Q2	**3C, 2C** •[1] centre $A = (-3, -2)$ •[2] centre $B = (3, 6)$ •[3] $P = (0, 2)$ •[4] $AB^2 = (3-(-3))^2 + (6-(-2))^2$ **OR** •[4] $\overrightarrow{AB} = \begin{pmatrix} 6 \\ 8 \end{pmatrix}$ •[5] $AB = 10$
05 P1 Q11	**1C, 5A** •[1] $(x-t)^2 + (y-0)^2 = 2^2$ •[2] $(x-t)^2 + (2x)^2 = 4$ •[3] $5x^2 - 2tx + t^2 - 4 = 0$ •[4] $"b^2 - 4ac" = 0$ •[5] $a = 5,\ b = -2t,\ c = t^2 - 4$ •[6] $4t^2 - 20(t^2 - 4) = 0$ $and\ \ t = \sqrt{5}$ **OR** $y = 2x \Rightarrow m_{tgt} = 2\ and\ m_{rad} = -\frac{1}{2}$ •[2] $equ\ of\ radius\ is\ x + 2y = t$ $ie\ x - t = -2y$ •[3] $(-2y)^2 + y^2 = 4$ •[4] $y = \frac{2}{\sqrt{5}}$ •[5] $x = \frac{1}{2}y \Rightarrow x = \frac{1}{\sqrt{5}}$ •[6] $t = x + 2y \Rightarrow t = \sqrt{5}$
05 P2 Q3	**4C, 4C, 4C**

	•[1] $m_{AB} = 1$ •[2] $m_{\perp} = -1$ •[3] midpoint $= (3,2)$ •[4] $y - 2 = -1(x-3)$ ••• •[5] $y = -\frac{1}{3}x......$ •[6] $m_{tgt} = -\frac{1}{3}$ •[7] $m_{rad} = 3$ •[8] $y - 0 = 3(x-1)$ •[9] $use\ x + y = 5$ ••• $y = 3x - 3$ •[10] $x = 2,\ y = 3$ •[11] $(x-2)^2 + (y-3)^2 = r^2$ •[12] $r^2 = 10$ **OR** •[9] D=(3,6) where D is intersection of the perp. to AB through B and the circle. •[10] C = midpoint of AD = (2,3)
06 P1 Q2	**2C, 4C** $(x-a)^2 + (y-b)^2 = r^2$ •[1] $(x-(-2))^2 + (y-3)^2$ •[2] $r^2 = 18$ •[3] $\mathbf{Q} = (-5,0)$ •[4] $m_{\text{diameter}} = 1$ stated or implied •[5] $m_{\text{tangent}} = -1$ •[6] $y - 0 = -(x-(-5))$ *For answers of the form* $x^2 + y^2 + 2gx + 2fy + c = 0$ •[1] $x^2 + y^2 + 4x - 6y + c = 0$ •[2] $c = -5$
06 P2 Q4	**5C** •[1] $C_1 = (3,4)$ •[2] $k = 6$ •[3] $R_1 = 5$ •[4] $R_2 = \sqrt{(-3)^2 + (-4)^2 - (-12)}$ **or equivalent** •[5] $\sqrt{37} > 5\ or$ "2nd circle" **OR** •[1] $x^2 + y^2 - 6x - 8y + 25 = 25$ •[2] $k = 6$ •[3] $R_1 = 5$ •[4] $R_2 = \sqrt{(-3)^2 + (-4)^2 - (-12)}$ **or equivalent** •[5] $\sqrt{37} > 5\ or$ "2nd circle"
07 P1	**5C**

<table>
<tr><td>Q5</td><td>

$\bullet^1$ $B = (7,8)$

$\bullet^2$ $r_{large} = \sqrt{7^2 + 8^2 - 77} = 6$

$\bullet^3$ $r_{small} = \frac{6}{3}$

$\bullet^4$ $D = (15,8)$

$\bullet^5$ $(x-15)^2 + (y-8)^2 = 2^2$

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<tr><td>07 P2
Q3</td><td>

6C

$\bullet^1$ $x^2 + (6-2x)^2 + 6x - 4(6-2x) - 7 = 0$

$\bullet^2$ $.....36 - 24x + 4x^2 .... - 24 + 8x....$

$\bullet^3$ $5x^2 - 10x + 5 = 0$

$\bullet^4$ $(x-1)^2 = 0$

$\bullet^5$ equal roots $\Rightarrow$ line is tangent

$\bullet^6$ $x = 1, \;\; y = 4$

OR

$\bullet^1$ $m_{line} = -2$

$\bullet^2$ $(-3,2)$ *and* $\frac{1}{2}$

$\bullet^3$ *equ. of radius* : $y - 2 = \frac{1}{2}(x+3)$

$\bullet^4$ $x = 1$

$\bullet^5$ $y = 4$

$\bullet^6$ check that $(1,4)$ lies on the circle

alternatives for $\bullet^4$ *and* $\bullet^5$

$\bullet^4$ $b^2 - 4ac = 0 \Rightarrow$ tangent

$\bullet^5$ $(-10)^2 - 4 \times 5 \times 5 = 0$

$\bullet^4$ use quad. formula to get roots

$\bullet^5$ equal roots $\Rightarrow$ line is tangent

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<tr><td>08 P2
Q4</td><td>

2C, 4C/B, 5B

$\bullet^1$ $(-4,-2)$

$\bullet^2$ $\sqrt{58}\;(\approx 7.6)$

$\bullet^3$ $(4,6)$ **and** $\sqrt{26}\;(\approx 5.1)$ *s / i* $\bullet^4$ *and* $\bullet^5$

$\bullet^4$ $d_{centres} = \sqrt{128}$ *accept* 11.3

$\bullet^5$ $\sqrt{58} + \sqrt{26}$ *accept* 12.7

$\bullet^6$ compare 12.7 and 11.3

$\bullet^7$ $x^2 + (4-x)^2 + \ldots$

$\bullet^8$ $x^2 + 16 - 8x + x^2 + \ldots$

$\bullet^9$ $2x^2 - 4x - 6 = 0$

		$\bullet^{10}$	$\bullet^{11}$
$\bullet^{10}$	x	3	−1
$\bullet^{11}$	y	1	5

alt. for $\bullet^7$ to $\bullet^{11}$:

$\bullet^7$ $(4-y)^2 + \ldots$

$\bullet^8$ $y^2 - 8y + 16 + y^2 + \ldots$

$\bullet^9$ $y^2 - 6y + 5 = 0$

		$\bullet^{10}$	$\bullet^{11}$
$\bullet^{10}$	y	1	5
$\bullet^{11}$	x	3	−1

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<tr><td>09 P2
Q4</td><td>

1C, 5C, 4A

</td></tr>
</table>

	•1 $(5+1)^2+(10-2)^2=100$	
	•2 $centre=(-1,2)$	
	•3 $Q=(-7,-6)$ **(no evidence requ.)**	
	•4 $m_{rad}=\frac{8}{6}$	
	•5 $m_{tgt}=-\frac{3}{4}$ **s / i by •6**	
	•6 $y-(-6)=-\frac{3}{4}(x-(-7))$	**Alternative for •8, •9 and •10**
	•7 $radius=20$ **s / i by •9 or •10**	
	•8 $centre=(5,10)$ **s / i by •9**	•8 $centre=(-19,-22)$ **s / i by •9**
	•9 $(x-5)^2+(y-10)^2=400$	•9 $(x+19)^2+(y+22)^2=400$
	•10 $(x+19)^2+(y+22)^2=400$	•10 $(x-5)^2+(y-10)^2=400$
10 P2 Q3	substitute	•1 $x^2+(3-x)^2+14x+4(3-x)-19=0$
		Method 1 : Factorising
	express in standard form	•2 $2x^2+4x+2$ $\}=0$ *see note 1*
	start proof	•3 $2(x+1)(x+1)$
	complete proof	•4 equal roots so line is a tangent
		Method 2 : Discriminant
		•2 $2x^2+4x+2=0$ *stated*
		•3 $4^2-4\times2\times2$
		•4 $b^2-4ac=0$ so line is a tangent
	coordinates of P	•5 $x=-1,\ y=4$
	: via centre and radius	**Method 1** : via centre and radius
	state centre of larger circle	•6 $(-7,-2)$ *see note 11*
	find radius of larger circle	•7 $\sqrt{72}$ *see note 6* *stated*
	find radius of smaller circle	•8 $\sqrt{8}$ *see note 7*
	strategy for finding centre	•9 e.g "Stepping out"
	interpret centre of smaller circle	•10 (1, 6)
	state equation	•11 $(x-1)^2+(y-6)^2=8$
		or $x^2+y^2-2x-12y+29=0$
	: via ratios	**Method 2** : via ratios
	state centre of larger circle	•6 $(-7,-2)$ *see note*
	strategy for finding centre	•7 e.g. "Stepping out"
	state centre of smaller circle	•8 (1, 6)
	strategy for finding radius	•9 $\sqrt{2^2+2^2}$
	find radius of smaller circle	•10 $\sqrt{8}$ *see note*
	state equation	•11 $(x-1)^2+(y-6)^2=8$

11 P2 Q7	state centre of C_1	$\bullet^1$ $(-1,\ 1)$	
	state radius of C_1	$\bullet^2$ 11	Do not accept $\sqrt{121}$
	state centre of C_2	$\bullet^3$ $(2,\ -3)$	
	find radius of C_2 in terms of p	$\bullet^4$ $\sqrt{13-p}$	Accept c in lieu of p
	interpret upper bound for p	$\bullet^5$ $p<13$	
	find distance between centres (d)	$\bullet^6$ 5	**stated explicitly**
	identify relevant relationship	$\bullet^7$ $\sqrt{13-p}<6$ or $r_2+d<11$ or $r_2<6$	
	develop relationship by squaring	$\bullet^8$ $13-p<36$	
	find lower bound for p	$\bullet^9$ $p>-23$	
12 P2 Q2		Substituting for y	
	rearrange linear equation	$\bullet^1$ $y=2x+5$	**stated, or implied by** $\bullet^2$
	substitute into circle	$\bullet^2$ $\ldots(2x+5)^2\ldots-2(2x+5)\ldots$	
	express in standard form	$\bullet^3$ $5x^2+10x-15$	$=0$ must appear at the $\bullet^3$ or $\bullet^4$ stage to gain $\bullet^3$.
	start to solve	$\bullet^4$ e.g. $5(x+3)(x-1)$	
	state roots	$\bullet^5$ $x=-3$ and $x=1$	
	determine corresponding y-coord	$\bullet^6$ $y=-1$ and $y=7$	
		Substituting for x	
		$\bullet^1$ $x=\dfrac{y-5}{2}$	**stated, or implied by** $\bullet^2$
		$\bullet^2$ $\left(\dfrac{y-5}{2}\right)^2\ldots-6\left(\dfrac{y-5}{2}\right)\ldots$	
		$\bullet^3$ $5y^2-30y-35$	$=0$ must appear at the $\bullet^3$ or $\bullet^4$ stage to gain $\bullet^3$.
		$\bullet^4$ e.g. $5(y+1)(y-7)$	
		$\bullet^5$ $y=-1$ and $y=7$	
		$\bullet^6$ $x=-3$ and $x=1$	
	centre of original circle	$\bullet^7$ $(3,\ 1)$	
	radius of original circle	$\bullet^8$ $\sqrt{40}$	Accept $r^2=40$
	Method 1 : Using midpoint	Method 1 : Using midpoint	
	$\bullet^9$ ss midpoint of chord	$\bullet^9$ $(-1,\ 3)$	
	$\bullet^{10}$ ss evidence for finding new centre	$\bullet^{10}$ e.g. stepping out or midpoint formula	
	$\bullet^{11}$ ic centre of new circle	$\bullet^{11}$ $(-5,\ 5)$	
	$\bullet^{12}$ ic equation of new circle	$\bullet^{12}$ $(x+5)^2+(y-5)^2=40$	
	Method 2 : Stepping out using P and Q	Method 2 : Stepping out using P and Q	
	$\bullet^9$ ss evidence of C_1 to P **or** C_1 to Q	$\bullet^9$ e.g. stepping out or vector approach	
	$\bullet^{10}$ ss evidence of Q to C_2 **or** P to C_2	$\bullet^{10}$ e.g. stepping out or vector approach	
	$\bullet^{11}$ ic centre of new circle	$\bullet^{11}$ $(-5,\ 5)$	
	$\bullet^{12}$ ic equation of new circle	$\bullet^{12}$ $(x+5)^2+(y-5)^2=40$	
13 P1 Q22	state centre	$\bullet^1$ $(-1,-2)$	
	find radius	$\bullet^2$ $\sqrt{32}$	

	find m_{radius}	•3 1
	state m_{tangent}	•4 -1
	state equation of tangent	•5 $y-2=-1\,(x-3)$
	find radius	•6 $\sqrt{8}$ **stated or implied by •7**
	state equation of circle	•7 $(x-10)^2+(y+1)^2=\left(\sqrt{8}\right)^2$
	expand and complete	•8 $x^2-20x+100+y^2+2y+1=8$ and complete **Accept** $2g=-20,\ 2f=2$ ¦ Centre $(10,-1)$ $g=-10, f=1$ ¦ $g=-10,\ f=1$ •6 Centre $(10,-1)$ ¦ $2g=-20,\ 2f=2$ •7 $r=\sqrt{(-10)^2+1^2-93}=\sqrt{8}$ •8 $\sqrt{32}=2\sqrt{8}$ $\frac{1}{2}\times\sqrt{32}=\frac{1}{2}\times 2\sqrt{8}=\sqrt{8}=$ radius of C_2
		Method 1
	substitute $y=5-x$ (or $x=5-y$)	Substituting for y •9 $x^2+(5-x)^2-20x+2(5-x)+93$
	express in standard quadratic form	•10 $2x^2-32x+128=0$
	start proof complete proof	•11 $2(x-8)^2=0$ •12 equal roots $\Rightarrow$ tangent ¦ •11 $(-32)^2-4\times2\times128$ •12 $b^2-4ac=0$ $\Rightarrow$ tangent
		or
		Substituting for x •9 $(5-y)^2+y^2-20(5-y)+2y+93=0$ •10 $2y^2+12y+18=0$
		•11 $2(y+3)^2=0$ •12 equal roots $\Rightarrow$ tangent ¦ •11 $12^2-4\times2\times18$ •12 $b^2-4ac=0$ $\Rightarrow$ tangent

		Method 2
	uses perpendicular gradients	•[9] m given line $= -1$, leading to $m_{radius} = 1$
	find equation of radius	•[10] $y + 1 = 1(x - 10)$
	starts proof	•[11] $y = -x + 5$ $y = x - 11$ $\Rightarrow x = 8$ $y = -3$
	completes proof	•[12] $(8)^2 + (-3)^2 - 20 \times (8) + 2(-3) + 93$ and complete
14 P1 Q23	substitute $3x - 5$	•[1] $x^2 + (3x-5)^2 + 2x - 4(3x-5) - 15 = 0$
	express in standard quadratic form	•[2] $10x^2 - 40x + 30 = 0$
	find x-coordinates	•[3] $x = 1$ | $x = 3$
	find y-coordinates	•[4] $y = -2$ | $y = 4$ •[3] •[4]
	state centre	•[5] $(-1,2)$
	calculate gradients	•[6] $m = -2$, $m = \frac{1}{2}$
	communicate result	•[7] demonstrates $m_1 \times m_2 = -2 \times \frac{1}{2} = -1$ $\Rightarrow$ PT is perpendicular to QT [or other appropriate statement]
	knows to find and states centre	•[8] centre $(2, 1)$
	calculate radius	•[9] radius $= \sqrt{10}$
	state equation of circle	•[10] $(x-2)^2 + (y-1)^2 = 10$
14 P2 Q8	correct values	•[1] $g = -p$, $f = -2p$, $c = 3p + 2$
	substitute and rearrange	•[2] $5p^2 - 3p - 2$
	knowing condition	•[3] $g^2 + f^2 - c > 0$
	factorise and solve	•[4] $(5p+2)(p-1) = 0 \Rightarrow p = -\frac{2}{5}, p = 1$
	correct range	•[5] $p < -\frac{2}{5}$, $p > 1$

Pre 2000 - Basic/General Equation

1	$\bullet^1$ $r^2 = 25$ stated or implied by $\bullet^2$. $\bullet^2$ $(x+3)^2 + (y-4)^2 = 25$
2	$\bullet^1$ $(1,2)$ $\bullet^2$ $\sqrt{(4-1)^2 + (5-2)^2}$ or equiv. $\bullet^3$ $(x-1)^2 + (y-2)^2 = 18$ or equiv.
3	$\bullet^1$ $radius_A = 2$ $\bullet^2$ $centre_A = (2,2)$ $\bullet^3$ $centre_B = (10,6)$ $\bullet^4$ $(x-10)^2 + (y-6)^2 = 4$
4	$\bullet^1$ use P as midpoint of C_1C_2 $\bullet^2$ $C_1 = (5,2)$ $\bullet^3$ $C_2 = (13,4)$ $\bullet^4$ radius $= \sqrt{17}$ $\bullet^5$ $(x-13)^2 + (y-4)^2 = 17$
5	$\bullet^1$ *strat*: *e.g.* origin to centre – radius $\bullet^2$ centre $= (4,3)$ $\bullet^3$ radius $= 2$ units $\bullet^4$ origin to centre $= 5$ units $\bullet^5$ $45m$
6	$\bullet^1$ centre of body $= (5,6)$ $\bullet^2$ radius of body $= 4$ $\bullet^3$ radius of head $= 3$ $\bullet^4$ centre of head $= (5,13)$ $\bullet^5$ $(x-5)^2 + (y-13)^2 = 9$
7	**(a)** $\bullet^1$ $A = \left(\frac{1}{2}, 3\right)$ $\bullet^2$ $r^2 = \frac{9}{4} + 1$ ***or*** $d^2 = 13$ $\bullet^3$ $\left(x - \frac{1}{2}\right)^2 + (y-3)^2 = \frac{13}{4}$ ***or*** $x^2 + y^2 - x - 6y + 6 = 0$ **(b)** $\bullet^4$ $B\ (8,8)$ $\bullet^5$ $F\ (14,12)$ $\bullet^6$ C $\left(\frac{13}{2}, 7\right)$ **(c)** $\bullet^7$ $\frac{1}{2}\pi DF + \frac{1}{2}\pi DE + \frac{1}{2}\pi EF$ $\bullet^8$ $\frac{1}{2}\pi DF = \frac{5}{2}\pi\sqrt{13}$ ***OR*** $\frac{1}{2}\pi EF = 2\pi\sqrt{13}$ $\bullet^9$ $\frac{5}{2}\pi\sqrt{13} + \frac{1}{2}\pi\sqrt{13} + 2\pi\sqrt{13}$

Pre 2000 - Using r = √(g² + f² – c)

1	$\bullet^1$ $g^2+f^2-c=-1\frac{3}{4}$ $\bullet^2$ $r=\sqrt{-1\frac{3}{4}}$ which is not possible
2	$\bullet^1$ $g^2+f^2-c>0$ $\bullet^2$ $r^2=9+4-c$ $\bullet^3$ $c<13$

Pre 2000 - Using Perpendicular Bisectors to find the centre

1	(a) $\bullet^1$ $m_{BC}=-\frac{1}{3}$ $\bullet^2$ $m_{\perp}=3$ $\bullet^3$ midpoint$_{BC}=\left(\frac{9}{2},\frac{5}{2}\right)$ $\bullet^4$ $y-\frac{5}{2}=3\left(x-\frac{9}{2}\right)$ (b) $\bullet^5$ $y-3x=-11$ $\bullet^6$ perp. bisector passes thr' centre **stated explicitly** $\bullet^7$ using $y-3x=-11$ **and** $y+2x=4$ $\bullet^8$ $(3,-2)$ $\bullet^9$ $r^2=25$ $\bullet^{10}$ $(x-3)^2+(y+2)^2=25$

Pre 2000 - Touching Circles and Distance between Circles

1	**(*a*)** $\bullet^1$ centre (0, 3) $\bullet^2$ centre (14, 10) $\bullet^3$ distance between centres = √245 $\bullet^4$ radius = 3 $\bullet^5$ radius = 10 $\bullet^6$ strategy (clearance = distance between centres minus sum of radii) $\bullet^7$ √245 – 13 $\bullet^8$ 133 mm or equivalent	**(*b*)** $\bullet^9$ $m_{PB}=1$ $\bullet^{10}$ $m_{AC}=-1$ $\bullet^{11}$ $y-3=-(x-7)$ for AC $\bullet^{12}$ strategy: substitute $\bullet^{13}$ substituting correctly $\bullet^{14}$ *eg* $2x^2-28x+96=0$ $\bullet^{15}$ $x=6,\ 8$ $(or\ \ y=2,4)$ $\bullet^{16}$ $(6,4)$ **and** $(8,2)$

Pre 2000 - Intersection of Lines and Circles

1	$\bullet^1$ $x^2+x^2-6x-2x-24=0$ $\bullet^2$ $(x+2)(x-6)=0$ $\bullet^3$ $(-2,-2)$ and $(6,6)$	***OR***	$\bullet^4$ centre is $(2,2)$ $\bullet^5$ radius is $\sqrt{32}$ or equivalent $\bullet^6$ $(x-2)^2+(y-2)^2=32$

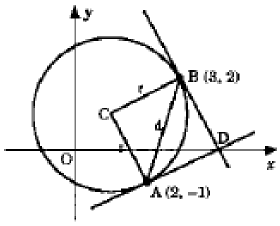
2	(*a*) $\bullet^1$ know to substitute $\bullet^2$ correct substitution $\bullet^3$ a "quadratic" = 0 $\bullet^4$ $x = -3,\ 1$ $\bullet^5$ $y - -5,\ 3$ (*b*) $\bullet^6$ $m_{\text{diameter}} = 2$ $\bullet^7$ $m_{\text{perpendicular}} = -\frac{1}{2}$ $\bullet^8$ centre $= (-1,\ -1)$ $\bullet^9$ equation: $y + 1 = -\frac{1}{2}(x+1)$

Pre 2000 - Tangency

1	$\bullet^1$ $x^2 + (x-k)^2 = 18$ $\bullet^2$ $2x^2 - 2kx + k^2 - 18 = 0$ $\bullet^3$ *strat*: "$b^2 - 4ac$" $= 0$ $\bullet^4$ $(-2k)^2 - 4.2.(k^2 - 18)$ $\bullet^5$ $k = \pm 6$
2	$\bullet^1$ sketch with point of con.P $= (3,-1)$ $\bullet^2$ Centre $C = (3,y)$ $\bullet^3$ $CO^2 = CP^2$ $\bullet^4$ $y = 4$ $\bullet^5$ *radius* $= 5$ $\bullet^6$ $(x-3)^2 + (y-4)^2 = 25$ ***OR*** $\bullet^1$ $(x-3)^2 + (y-k)^2 = r^2$ $\bullet^2$ $r^2 = k^2 + 9$ $\bullet^3$ $(x-3)^2 + (-1-k)^2 = r^2$ has '=' roots $\bullet^4$ reduce to $x^2 - 6x + (2k+1)$ $\bullet^5$ $k = 4$ $\bullet^6$ $(x-3)^2 + (y-4)^2 = 25$ ***OR*** $\bullet^1$ sketch with point of con.P $= (3,-1)$ $\bullet^2$ $x^2 + y^2 + 2gx + 2fy + c = 0$ $\bullet^3$ $(0,0) \Rightarrow c = 0$ $\bullet^4$ $(6,0) \Rightarrow g = -3$ $\bullet^5$ $(3,-1) \Rightarrow f = -4$ $\bullet^6$ $x^2 + y^2 - 6x - 8y = 0$
3	(*a*) $\bullet^1$ centre $= (4,-1)$ $\bullet^2$ inner radius $= 5$ $\bullet^3$ $(x-4)^2 + (y+1)^2 = 25$ $\bullet^4$ completing argument (*b*) $\bullet^5$ *e.g.* $x = \frac{4}{3}y - 3$ $\bullet^6$ $\left(\frac{4}{3}y - 3\right)^2 + y^2 - 8\left(\frac{4}{3}y - 3\right) + 2y - 8 = 0$ $\bullet^7$ $\frac{16}{9}y^2 - 8y + 9 + y^2 - \frac{32}{3}y + 24 + 2y - 8$ $\bullet^8$ $y^2 - 6y + 9 = 0$ $\bullet^9$ *e.g.* $(y-3)(y-3) = 0$ $\bullet^{10}$ equal roots $\Rightarrow$ line is a tangent $\bullet^{11}$ $(1,3)$

4	**(a)** $\bullet^1$ Strategy: know to find m $\bullet^2$ $m = \frac{4}{3}$ $\bullet^3$ $y + 46 = \frac{4}{3}(x-3)$ **(b)** $\bullet^4$ $x^2 + y^2 = 900$ or equivalent **(c)** $\bullet^5$ Strategy: know to substitute $\bullet^6$ $x^2 + \left(\frac{4}{3}x - 50\right)^2 = 900$ $\bullet^7$ $(x-24)^2$ or evaluate the discriminant $\bullet^8$ communication relating to tangency $\bullet^9$ $(24, -18)$

Pre 2000 - Equations of Tangents

1	$\bullet^1$ $centre = (1,1)$ $\bullet^2$ $m_{radii} = \frac{1}{2},\ -2$ $\bullet^3$ $m_{tgts} = -2,\ \frac{1}{2}$ $\bullet^4$ $-2 \times \frac{1}{2} = -1 \Rightarrow$ tgts are $\perp$ *OR* $\bullet^1$ $centre = (1,1)$ $\bullet^2$ $r = \sqrt{5},\ d = \sqrt{10}$ $\bullet^3$ Show $A\hat{C}B = 90^\circ$ $\bullet^4$ State tangents $\perp$ to radii 
2	$\bullet^1$ Centre $= (3,-2)$ $\bullet^2$ $m_{rad} = \frac{4}{3}$ $\bullet^3$ $m_{tgt} = -\frac{3}{4}$ $\bullet^4$ $y - 2 = -\frac{3}{4}(x-6)$
3	$\bullet^1$ centre $= (-1,2)$ $\bullet^2$ $m_{radius} = \frac{1}{2}$ $\bullet^3$ $m_{tgt} = -2$ $\bullet^4$ $y - 4 = -2(x-3)$
4	$\bullet^1$ *strat*: use centre and tgt $\perp$ radius $\bullet^2$ $centre = (2,-3)$ $\bullet^3$ $m_{radius} = 4$ $\bullet^4$ $m_{tgt} = -\frac{1}{4}$ $\bullet^5$ $y - 1 = -\frac{1}{4}(x-3)$
5	$\bullet^1$ *strat*: *i.e* find AC then AB $\bullet^2$ centre $= (2,2)$ **and** radius $= 5$ $\bullet^3$ $AC = 10$ $\bullet^4$ $AB = \sqrt{75}$ units $\bullet^5$ area $= \frac{25}{2}\sqrt{3}$ square units

Mixed Questions

1	(a) •1 $centre = (-3,-2)$ •2 $m_{rad} = \frac{1}{2}$ •3 $m_{tgt} = -2$ •4 $y-(-1) = -2(x-(-1))$ (b) •5 $B = (0,-3)$ (c) •6 $C = (-2,1)$ (d) •7 $r^2 = 5$ •8 $(x+1)^2 + (y+1)^2 = 5$
2	•1 radius of inner circle $= \frac{3}{2}$ •2 centres are $\left(0,-1\frac{1}{2}\right)$ **and** $(0,-2)$ •3 circumferences are 3π **and** 4π •4 *e.g.* $\text{tgt}^2 = 4^2 - 2^2$ •5 $\text{tgt} = \sqrt{12}$ •6 29
3	**(a)** •1 $m_l = 2$ •2 $m_r = -\frac{1}{2}$ •3 $y+1 = -\frac{1}{2}(x-4)$ **(b)** •4 $(x-4)^2 + (y+1)^2 = r^2$ •5 $(x-4)^2 + (2x+7)^2 = r^2$ •6 $5x^2 + 20x + (65 - r^2) = 0$ •7 $\Delta = 400 - 4 \times 5(65 - r^2) = 0$ •8 $r^2 = 45$

4	**(a)**	•1 $y = 2x$ •2 $y = 2x^2$ •3 $x^2 + y^2 = 5$
	(b)	•4 $y = 2x - 4$ •5 $y = -2x^2$ •6 $centre = (2,0)$ •7 $(x-2)^2 + y^2 = 5$
	(c)	•8 *show* $(2,0)$ *satisfies* $y = 2x - 4$ •9 $2x - 4 = -2x^2$ •10 $(x+2)(x-1) = 0$ •11 $(-2,-8)$, $(1,-2)$
5	**(a)**	•1 $\frac{dy}{dx} = \frac{2}{9}x$ choose 1 point e.g. (3,7) •2 *parabola*: $m_{tgt\ x=3} = \frac{2}{3}$ •3 circle centre $= (0,5)$ •4 *circle*: $m_{rad\ x=3} = \frac{2}{3}$, $m_{tgt\ x=3} = -\frac{3}{2}$ •5 $\left(m_{tgt(P)\ x=3}\right) \times \left(m_{tgt(C)\ x=3}\right) = -1$ *so "curves orthogonal"* •6 deal totally with other point
	(b)	•7 $y - 7 = \frac{3}{2}(x+3)$ •8 circle centre $= \left(0, 11\frac{1}{2}\right)$ •9 $r^2 = \frac{117}{4}$ •10 $x^2 + \left(y - 11\frac{1}{2}\right)^2 = \frac{117}{4}$